QUESTIONS & ALTERNATIVES

Alejandro Pérez Carballo

University of Massachusetts, Amherst
apc@umass.edu

Recall: one problem for the theory of Sauerland 2004.

There is still another problem we will not deal with here, viz. that it predicts that

> Seth took the dancing class or the juggling class or both.

implicates that Seth didn't take both the dancing class and the juggling class.

(1) a. Every guest liked some of the dishes.
  b. $\rightsquigarrow$ Not every guest liked all of the dishes.
  c. $\rightsquigarrow$ Every guest liked some but not all of the dishes.

Sauerland's framework cannot predict that (1c) is an implicature of (1a).
Recall that

(2) Some guest liked all of the dishes.

is, on Sauerland's approach, a scalar alternative to (1a). One possible solution to the problem illustrated in (2) would be to say that all alternatives that are *not weaker* than the assertion give rise to a secondary implicature. But why would something not more informative than the uttered sentence could give rise to a primary (let alone a secondary) implicature?
Another possible solution would be to postulate that

(3) Every guest liked some of the dishes and some guest liked all of the dishes.

is a scalar alternative to (1a). But it is not obvious how to modify Sauerland's approach to get this result in a non-*ad hoc* way.
One of the upshots of Spector 2007 is that there is a well-motivated neo-Gricean framework, different from Sauerland's, on which (3) is an alternative to an utterance of (1a). Moreover, on this alternative picture, the Gricean reasoning does not require access to the syntactic structure of the uttered sentence.

This suggests that it will not be able to solve the 'or both' problem mentioned in the previous margin note. But perhaps that is a problem that requires an alternative treatment?

QUESTIONS & GRICEAN REASONING

Think of an utterance of a sentence φ as taking place as an answer to a (perhaps implicit) question Q. Suppose we can characterize a set of belief states $I_Q(\varphi)$ such that:

(4) if a speaker's belief state is among those in $I_Q(\varphi)$, φ is a 'best answer' to Q.

Then we can recast the Gricean reasoning as follows:

(5) The speaker uttered φ in response to Q. Therefore, her belief state

must be in $I_Q(\varphi)$.

We can draw an even stronger conclusion if we assume that the speaker is as well-informed with respect to Q as possible, given the answer she gave:

(6) The speaker uttered φ in response to Q. Therefore, her belief state must be among those in $I_Q(\varphi)$ that are best-informed with respect to Q.

The proposal in Spector 2007 is essentially a way of cashing out this intuitive picture.

We will think of a *question* as a finite collection of propositions, understood as subsets of a fixed set of possible worlds $\mathcal{W}$.

For now, we will not be concerned with the issue of whether questions, thus understood, give the meaning of interrogative sentences, nor with the question of how to assign questions to interrogatives. For a sample of some of the issues, see Groenendijk and Stokhof 1984, 1997; Hamblin 1958, 1973; Karttunen 1977.

(7) For every question Q, we will call $\mathcal{B}(Q)$ the smallest Boolean algebra containing all the propositions in Q.

(8) For every question Q, we will call π_Q the collection of *atoms* of $\mathcal{B}(Q)$—i.e. the collection of non-empty propositions in $\mathcal{B}(Q)$ that are minimal elements under the partial order induced by the subset relation.

To see that π_Q will be a partition, note that $\bigcup \pi_Q$ must equal $\mathcal{W}$, since every $w \in \mathcal{W}$ must be in some element of $\mathcal{B}(Q)$. Also note that for any two $x \neq y \in \pi_Q$, $x \cap y = \varnothing$, for otherwise we'd have $x \cap y \subsetneq x$ or $x \cap y \subsetneq y$, with $x \cap y \in \mathcal{B}(Q)$, contrary to the assumption that x and y are minimal.

Every question can thus be associated with a *partition* of $\mathcal{W}$: a collection of pairwise disjoint and jointly exhaustive propositions. This assignment, however, will not be one-to-one: the same partition may be generated by multiple questions.

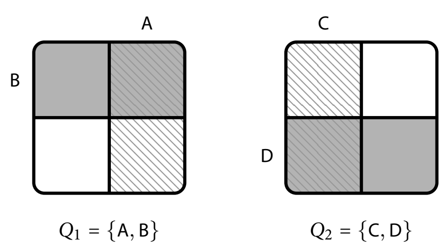


Figure 1: Two different questions that generate the same partition. The strongly relevant propositions with respect to these two questions are the same, viz. those propositions that do can be obtained as unions of the corresponding atoms.

Following Spector, we will assume that the speaker will always express a *strongly relevant* proposition in the following sense:

It would be interesting to explore what would happen if we drop this assumption and build it instead into the definition of an optimal answer to the question under discussion. This would probably allow us to use Gricean reasoning to draw inferences about what the speaker takes the question under discussion to be.

(9) A proposition $X \subseteq \mathcal{W}$ is strongly relevant with respect to Q iff $X \neq \mathcal{W}$ and $X \in \mathcal{B}(Q)$.

We will also think of belief states as essentially strongly relevant propositions.

Alternatively, we can think of belief states as propositions (or *information states*) and say we will only be interested in differences between belief states that bear on the question under discussion.

(10) An utterance of a sentence φ in is a best answer to Q in c by S (relative to a set of alternatives $\mathcal{A}$) iff S's belief state B_S entails $[\![\varphi]\!]_c$ and for every $X \in \mathcal{A}$, if X is entailed by B_S then $X \notin [\![\varphi]\!]_c$.

For each φ and Q (and c), let $I_Q(\mathcal{A}, \varphi)$ denote the collection of belief states such that φ is a best answer to Q in c relative to that belief state and alternatives A.

(11) $M_Q(\mathcal{A}, \varphi)$ is the collection of belief states in $I_Q(A, \varphi)$ that are not weaker than any other element of $I_Q(\mathcal{A}, \varphi)$.

The schematic version of Gricean reasoning can now be put thus:

(12) The speaker uttered φ as an answer to Q. The alternatives were those in A. Therefore, her belief state must be in $M_Q(\mathcal{A}, \varphi)$.

The goal of Spector 2007 is to show that, for at least a particular kind of answers to a question, this Gricean reasoning can emulate the predictions of the exhaustivity approach to scalar implicatures.

POSITIVE ANSWERS & THEIR ALTERNATIVES

If X is a proposition, we will let X^c denote its complement. We will sometimes write $X^\varnothing$ to denote X itself. Let $n = |Q|$ and let us fix an enumeration $\{A_i : 0 \leq i < n\}$ of the elements of Q. Note that:

(13) Every element of π_Q is of the form

$$\bigcap_{i=0}^{n-1} A_i^{j_i},$$

where for each $0 \leq i < n$, $j_i \in \{\varnothing, c\}$.

We will say that

(14) a proposition is *Q-positive* iff it is in the closure of Q under unions and intersections.

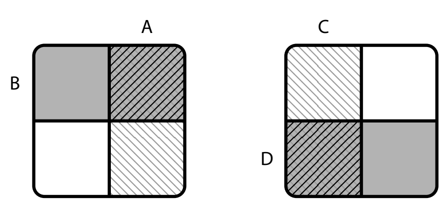


Figure 2: On the left, the Q_1-positive propositions (three of them). On the right, the Q_2-positive ones (also, three of them).

It can be shown that

(15) a proposition is Q-positive iff it is the union of intersections of elements of Q.

It can also be shown that

This is essentially the definition Spector 2007 starts out from.

(16) A proposition X is Q-positive iff whenever

$$\bigcap_{i=0}^{n-1} A_i^{j_i} \subseteq X \cap A_{i^*}^c,$$

for some $i^* < n$,

$$A_{i^*} \cap \bigcap_{i \neq i^*} A_i^{j_i} \subseteq X.$$

Intuitively: a proposition X is positive iff for each i, if $w \in X \cap A_i^c$, then $f_i(w) \in X \cap A_i^c$, where $f_i(w)$ is the 'closest' world to w in which A_i is true.

The notion of closeness relevant here is this: for each i, if $w \in A_i$, then $f_i(w) = w$, else, $f_i(w)$ is a world in A_i which agrees with w in the truth-value of each A_j with $j \neq i$. Using this notion of closeness, we can define a Q-positive proposition as follows: X is Q-positive iff for all i, if $w \in A_i^c \cap X$, then $w \in A_i \Box\!\!\rightarrow X$. In other words: there is no X-world in which the truth of of an element of Q would have made X false.

Let us make the following stipulation:

(17) If $[\![\varphi]\!]_c$ is a Q-positive proposition, then the set of alternatives to φ (in c) in response to a question Q, which we denote by $\mathcal{A}_c(\varphi, Q)$, is the set of all Q-positive propositions.

Given (17), we can apply the reasoning in (12) to conclude that:

(18) If a speaker S utters a sentence that expresses a Q-positive proposition X, one can infer that his belief state does not entail any Q-positive proposition that is stronger than X.

AN EXAMPLE

Go back to (1a), which I repeat below for convenience:

(1a) Every guest liked some of the dishes.

and assume the guests are Alice, Beth, and Carol, and the dishes are a main course and a dessert. Assume further the question under discussion is *Who liked which dish?*, which we will identify with the set Q_d of propositions expressed by sentences of the form ⌜α liked the main course⌝ or ⌜α liked the dessert⌝.

From (17), we know that the Q_d-positive propositions are the result of closing Q_d under conjunction and disjunction. In particular:

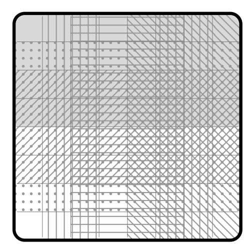


Figure 3: The partition corresponding to Q_d contains 64 atoms. Think of the different patterns as highlighting each of the different propositions in Q_d. The positive propositions are those strongly Q_d-relevant X with the following property: if an atom is included in X, for any pattern it does not include there is one that is just like the atom but with the missing pattern that is also in X.

(19) (Alice liked the main course or the dessert) and (Beth liked the main course or the dessert) and (Carol liked the main course or the dessert).

and

(20) (Alice liked the main course and the dessert) or (Beth liked the main course and the dessert) or (Carol liked the main course and the dessert).

are Q_d-positive propositions. Note that (19) is contextually equivalent to (1a), and (20) is contextually equivalent to (2):

(2) Some guest liked all of the dishes.

Now, let $\mathcal{A}^+$ denote the collection of Q_d-positive propositions. We can now see that no belief state in $M_{Q_d}(\mathcal{A}^c, (1a))$ entails $[\![(2)]\!]_c$. For,

(21) If B_S entails $[\![(20)]\!]_c =_c [\![(2)]\!]_c$, then there is a positive proposition, viz. the conjunction of $[\![(20)]\!]_c =_c [\![(2)]\!]_c$ and $[\![(19)]\!]_c =_c [\![(1a)]\!]_c$, which is stronger than $[\![(19)]\!]_c =_c [\![(1a)]\!]_c$ and entailed by B_S.

I write $A =_c B$ whenever A and B have the same intersection with the context set in c.

We can thus conclude that the speaker does not believe that (2) is true.

To get to the conclusion that (1a) implicates the *negation* of (2), we just need to show that

(22) any belief state in $I_{Q_d}(\mathcal{A}^+, (1a))$ that does not entail the negation of (2) is not in $M_{Q_d}(\mathcal{A}^+, (1a))$.

We will only show something that, strictly speaking, is weaker, viz.

(23) The conjunction of $[\![(1a)]\!]_c$ and the negation of $[\![(2)]\!]_c$ does not entail any Q_d positive proposition that is not entailed by all members of $I_{Q_d}(\mathcal{A}^+, (1a))$.

This is easiest to see if we forget about Carol. The resulting partition has the following form, with the gray region corresponding to $((A_m \cup A_d) \cap (B_m \cup B_d)) \cap ((A_m \cap A_d) \cup (B_m \cap B_d))^c$:

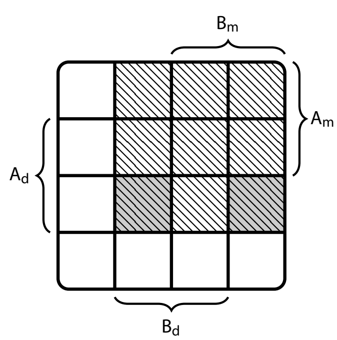


Figure 4: The area covered with northwest lines corresponds to $[\![(1a)]\!]_c$.

The weakest Q_d-positive propositions that are stronger than $[\![(1a)]\!]_c$ but which do not entail $[\![(2)]\!]_c$ are:

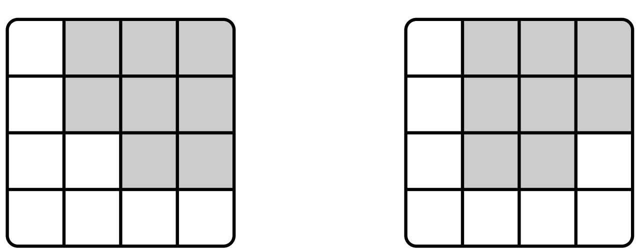


Figure 5: From left to right: $(A_m \cap B_d) \cup (A_m \cap B_m) \cup (A_d \cap B_m)$ and $(A_m \cap B_d) \cup (A_d \cap B_m) \cup (A_d \cap B_d)$.

And clearly, neither one of them is entailed by conjunction of $[\![(1a)]\!]_c$ and the negation of $[\![(2)]\!]_c$—that is, by the proposition consisting of the two gray cells in figure 4.

RECOVERING THE BENEFITS OF SAUERLAND'S APPROACH

Can this theory solve the symmetry problem and the disjunction problem?

Recall:

(24) a. Alice liked some of the dishes.
b. $\rightsquigarrow$ Alice did not like all of the dishes.
c. $\not\rightsquigarrow$ Alice liked all of the dishes.

(25) a. Alice liked the wine or some of the dishes.
b. $\rightsquigarrow$ Alice did not like all of the dishes.

(26) a. Alice liked the main course or Beth liked the main course or Carol liked the main course.
b. $\rightsquigarrow$ Exactly one of Alice, Beth, and Carol liked the main course.

To account for (24) we need to show that, for the relevant question Q_s,

(27) (24a) and (24b) are Q_s-positive, but (24c) is not.

This can be done if we stipulate that $Q_s = Q_d$, for

(28) Alice liked the main course or Alice liked the dessert.

and

(29) Alice liked the main course and Alice liked the dessert.

are Q_d-positive, but

Actually, we also need to show that we can strenghten a belief state to one which entails the negation of the relevant alternative we do not end up entailing any new positive proposition. I leave that as a (non-trivial!) exercise.

(30) (Alice liked the main course and Alice did not like the dessert) or (Alice did not like the main course and Alice liked the dessert).

is not.

To account for (25) we need to show that, for the relevant question Q_c, (25a) is Q_c positive, and so is

(31) Alice liked all of the dishes.

This too can be done if we assume that the question is *Who liked what?*, and the answers are of the form $\ulcorner\alpha$ liked the wine$\urcorner$, $\ulcorner\alpha$ liked the main course$\urcorner$, or $\ulcorner\alpha$ liked the dessert$\urcorner$. For then: (25a) is contextually equivalent to

(32) Alice liked the wine or (Alice liked the main course and the dessert).

and (31) is contextually equivalent to

(33) Alice liked the main course and the dessert.

Finally, in order to account for (26a) we need to be a bit more subtle. First, assume that the question under discussion is Q_d. Note then that each of the disjuncts are Q_d-positive propositions which are stronger than (26a):

(34) a. Alice liked the wine.
b. Alice liked the main course.
c. Alice liked the dessert.

We can conclude that, e.g.

(35) The speaker's belief state does not entail (34a).

We cannot, however, conclude that the speaker's belief state entails the negation of (34a). If it did, it would entail

(36) Alice liked the main course or the dessert.

And this is a Q_d-positive proposition that is stronger than (26a).

Note further that:

(37) (Alice liked the wine and the main course) or (Alice liked the main course and the dessert) or (Alice liked the wine and the dessert).

is a Q_d-positive proposition which is stronger than (26a). We can then conclude that

(38) The speaker's belief state does not entail (37).

This is not enough to get us (26b). For that, we need to show that if the speaker's belief state does not entail (26b), it is not in $I_{Q_d}(\mathcal{A}^+, (26a))$.

It helps to look at a picture:

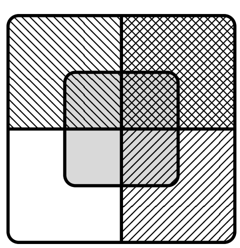


Figure 6: We can focus on the subset of Q_d consisting of the three propositions in (34). Each of the different patterns corresponds to one of them. The areas covered by at least two patterns correspond to the worlds in which (37) is true.

The area corresponding to (37) is the area covered by at least two of the patterns. It helps to highlight it:

Now suppose $X \in I_{Q_d}(\mathcal{A}^+, (26a))$. Suppose that X entails (26a) and does not entail (37). Using A_w, A_m, and A_d to abbreviate the disjuncts in (34) in the

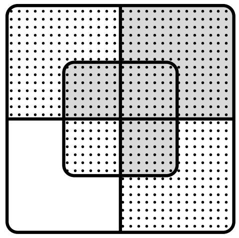


Figure 7: The grayed out area corresponds to (37). The area covered by dots corresponds to (26a).

natural way, we can see that:

(39) $X \cap ((A_w \cap A_m) \cup (A_m \cap A_d) \cup (A_w \cup A_d))^c = Y \in I_{Q_d}(\mathcal{A}^+, (26a))$.

To see why, look back at figure 7. Since X is in $I_{Q_d}(\mathcal{A}^+, (26a))$, X cannot entail any of the two-term disjunctions, so it must overlap with each of the non-gray dotted regions. So Y is non-empty, it entails $A_w \cup A_m \cup A_d$, and it entails the proposition we're interested in, viz.

$$((A_w \cap A_m) \cup (A_m \cap A_d) \cup (A_w \cup A_d))^c.$$

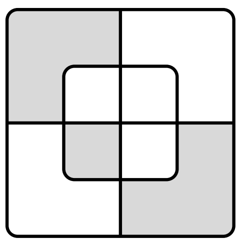
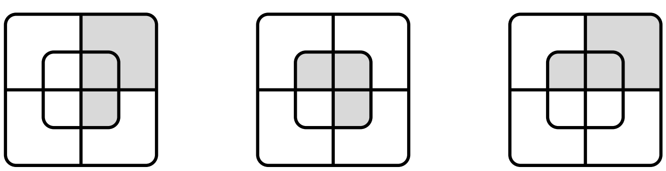


Figure 8: Since X is strongly relevant with respect to Q_d, Y will just be the proposition highlighted in gray here—in other words, the conjunction of $[\![(26a)]\!]_c$ and the negation of $[\![(37)]\!]_c$.

We just need to get our hands a bit dirty to show that Y does not entail any Q_d-positive proposition weaker than (34). It is clear that Y does not entail any of the disjuncts, and thus it does not entail their conjunction. In addition to those, there are only three interesting cases to check, viz. $(A_w \cap A_m) \cup (A_w \cap A_d)$, $(A_w \cap A_m) \cup (A_m \cap A_d)$, and $(A_w \cap A_d) \cup (A_m \cap A_d)$:



Since Y does not entail any of these disjunctions, it cannot entail any of the disjuncts. And since by construction it does not entail (37), we can conclude that the strongest Q_d-positive proposition entailed by Y is (26a).

REFERENCES

Groenendijk, J. And Stokhof, M. 1984. *Studies in the Semantics of Questions and the Pragmatics of Answers*. PhD thesis. Amsterdam: University of Amsterdam.

———. 1997. Questions. In: *Handbook of Logic and Language*. Ed. by J. van Benthem and A. ter Meulen. Amsterdam: North Holland, pp. 1055–1124.

Hamblin, C. L. 1958. Questions. *Australasian Journal of Philosophy* 36.3, pp. 159–168.

———. 1973. Questions in Montague English. *Foundations of Language* 10.1, pp. 41–53.

Karttunen, L. 1977. Syntax and Semantics of Questions. *Linguistics and Philosophy* 1.1, pp. 3–44.

Sauerland, U. 2004. Scalar Implicatures in Complex Sentences. *Linguistics and Philosophy* 27.3, pp. 367–391.

Spector, B. 2007. Scalar Implicatures: Exhaustivity and Gricean Reasoning. In: *Questions in Dynamic Semantics*. Ed. by M. Aloni, A. Butler, and P. Dekker. Bingley: Emerald, pp. 225–254.